Name: ______________________

Math 378 Spring 2011
Assignment 1
Solutions

Brualdi 3.4. Partition the set $\{1, 2, \ldots, 2n\}$ as $\{1, 2\}, \{3, 4\}, \{5, 6\}, \ldots, \{2n-1, 2n\}$. There are $n+1$ integers but only n sets, so we must choose two from the same set.

Brualdi 3.5. Partition the set as $\{1, 2, 3\}, \{4, 5, 6\}, \ldots, \{3n-2, 3n-1, 3n\}$. Again, n sets and $n+1$ choices so we must choose two from the same set.

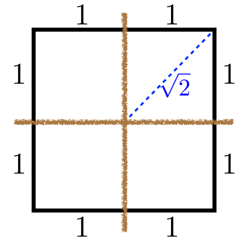
Brualdi 3.6. If $n+1$ distinct integers are chosen from $\{1, 2, \ldots, kn\}$, then there are two which differ by at most $k-1$.

Brualdi 3.7. Suppose we have 52 of them such that no difference is divisible by 100. Then they have distinct remainders when divided by 100. There are 100 possible such remainders. Partition them into 51 sets: $\{0\}, \{50\}, \{1, 99\}, \{2, 98\}, \{3, 97\}, \ldots, \{49, 51\}$. We must choose two from the same part and their sum is divisible by 100.

Brualdi 3.10. For each $i \in \{1, 2, \ldots, 49\}$, let a_i be the total number of hours watched in the first i days. Then $a_1, a_2, \ldots, a_49$ are distinct positive integers less than or equal to 77 and $a_1 + 20, a_2 + 20, \ldots, a_49 + 20$ are distinct and at most 97. So we have 98 integers from $\{1, 2, \ldots, 97\}$ which means two are equal. In other words, $a_i = a_j + 20$ for some i and j, so the child watches exactly 20 hours on days $j+1, j+2, \ldots, i$.

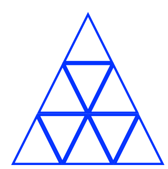
Brualdi 3.16. It seems the possible number of acquaintances is $\{0, 1, \ldots, n-1\}$ (n possibilities). However, if someone knows nobody, then no one knows $n-1$ people and vice-versa, so there are only $n-1$ possibilities. Thus, two people must know the same number.

Brualdi 3.17.



There are 5 points and 4 squares, so two points must go in the same square. The distance between those two points is at most $\sqrt{2}$.

Brualdi 3.18.



There are 10 points and 9 triangles, so two points must go in the same triangle. The distance between those two points is at most $\frac{1}{3}$.

Extra Problem 1. $r(2, 2, 2, 6) = 6$. This is because as soon as we use the first, second, or third color, we are done (ie. $K_6 \to K_2, K_2, K_2, K_6$). Also, $K_5 \not\to K_2, K_2, K_2, K_6$ since all edges could be the fourth color.

Extra Problem 2. Let $a_1, a_2, \ldots, a_N$ be the sequence and the vertices of a K_N. We color the edges as follows:

$$\text{For } i < j, \text{ color the edge } \begin{cases} \text{red}, & \text{if } a_i < a_j. \\ \text{blue}, & \text{if } a_i > a_j. \end{cases}$$

If $N = r(m, n)$, then there is either a red K_m or a blue K_n. A red K_m gives an increasing subsequence of length m, while a blue K_n gives a decreasing subsequence of length n.

Extra Problem 3. There are many ways. Here's one: $6, 5, 4, 3, 2, 1, 12, 11, 10, 9, 8, 7, \ldots,$ $36, 35, 34, 33, 32, 31$. Also, it's reversal or $6, 12, 18, 24, 30, 36, 5, 11, 17, 23, 29, 35, \ldots,$ $1, 7, 13, 19, 25, 31$.

Note that for all of these, if you form the sequence $m_1, m_2, m_3, \ldots, m_{36}$ where m_i is the length of a longest increasing subsequence with first term a_i, then each integer $1, 2, 3, 4, 5,$ and 6 appears precisely 6 times in the m_i sequence.