EXAM #2 2012-3-5

GUIDELINES: This Exam is being given under the guidelines of the **Honor Code**. You are expected to respect those guidelines and to report those who do not. Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. There are 6 questions for a total of 70 points.

Name: ____________________

1. Consider the statement "If n is a prime and $n > 2$, then n is odd" where $n \in \mathbb{N}$.

   (a) (8 points) Write the negation of the statement. Is the negation of the statement true or false? Provide a counterexample or some short explanation.

   (b) (8 points) Write the contrapositive of the statement. Is the contrapositive of the statement true or false? Provide a counterexample or some short explanation.

Points earned: $\square$ / 16

2. (12 points) Prove that the square of any odd integer j can be written in the form $8k + 1$ for some $k \in \mathbb{Z}$. In other words: "If j is an odd integer, there exists an integer k such that $j^2 = 8k + 1$."

3. (8 points) Let n be an integer. Use contraposition to prove that if n is not even, then $n + 1$ is not odd.

Points earned: $\boxed{}$ / 20

4. Express each of the following sums using summation notation.
   (a) (4 points) $4+5+6+7$

   (b) (5 points) $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\cdots+\frac{1}{20}$

   (c) (5 points) $5+9+13+17+21+\cdots+49$

5. (10 points) Prove that for $t \in \mathbb{N}$,

$$\left(\frac{t(t+1)}{2}\right)^2 + (t+1)^3 = \left(\frac{(t+1)\left[(t+1)+1\right]}{2}\right)^2.$$

(Hint: Do **not** use induction.)

Points earned: $\boxed{}$ /24

6. (10 points) Prove that $\sum_{i=1}^{n}(2i-1) = n^2$ for all $n \in \mathbb{N}$.

Points earned: $\boxed{}$ / 10