EXAM #1 2012-02-06

Guidelines: This Exam is being given under the guidelines of the **Honor Code**. You are expected to respect those guidelines and to report those who do not. Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. There are 5 questions for a total of 105 points.

Name: ______________________________

1. Consider $A = \{1,2,3,4,5\}$, $B = \{4,5,6\}$ and $C = \{1,2\}$ as subsets of $[9] = \{1,2,\ldots,9\}$.
   (a) (2 points) Is $2 \in A$? _____
   (b) (2 points) Is $2 \in B$? _____
   (c) (3 points) Is $2 \subseteq A$? _____
   (d) (3 points) Is $\{2\} \subseteq A$? _____
   (e) (3 points) Is $\emptyset \in A$? _____
   (f) (3 points) Is $\emptyset \subseteq A$? _____
   (g) (4 points) Find $A \cup B$.
   (h) (4 points) Find $A \cap B$.
   (i) (4 points) Find $B \cap C$.
   (j) (4 points) Find $A \setminus C$.
   (k) (4 points) Find $\mathcal{P}(C)$.
   (l) (4 points) Is $A \subseteq B$? Why or why not.
   (m) (4 points) Is $C \subseteq A$? Why or why not.
   (n) (6 points) Let $A \triangle B = (A \cup B) \setminus (A \cap B)$. This is called the ***symmetric difference*** of A and B. Find $A \triangle B$.

2. (a) (3 points) Let O be the set of all odd integers. Write, using set notation, the set of all odd integers.
   (b) (8 points) Write, using set notation, **two different representations** of a set that contains the quotients of any two odd integers.
   (c) (4 points) Is the set from part (b) equal to $\mathbb{Q}$? Why or why not.

Points earned: $\boxed{}$ / 65

3. (10 points) A fraternity has a rule for new members: each must always tell the truth or always lie. They know who does which. If I meet three of them on the street and they make the following statements, which ones (if any) should I believe? Explain your reasoning.

   A says: "All three of us are liars."

   B says: "Exactly two of us are liars."

   C says: "The other two are liars."

4. (10 points) Show that the logical expression S is equivalent to the logical expression $\sim S \implies (R \wedge \sim R)$.

5. Provide an argument or counterexample to support the truth value of the following statements.

   (a) (5 points) $\exists x \in \mathbb{R}, x^2 - x = 0$.

   (b) (5 points) $\forall x \in \mathbb{R}, \sqrt{x^2} = x$.

   (c) (5 points) $\exists x, y \in \mathbb{R}, x + y + 3 = 8$.

   (d) (5 points) $\forall x, y \in \mathbb{R}, x + y + 3 = 8$

Points earned: $\square$ /40