Name: ____________________________

Math 283 Spring 2012
Assignment 5

To Hand In:

D'Angelo & West Ch. 4: 2, 18, 20 (a and b), 21, 37, 40, 42, 43, 45, 46, 47, 49

Not To Hand In:

D'Angelo & West Ch. 4: 1, 4–12, 13, 20 (c), 23, 24, 29, 33, 36, 39, 50, 51

Extra Problems

1. Determine which cubic polynomials from $\mathbb{R}$ to $\mathbb{R}$ are injective.

(Hint: This is easy if calculus is allowed (but it's not!). To avoid calculus, first use geometric arguments to reduce the problem to the case $x^3 + rx$.)

(Hint 2: I wrote an article in *Mathematics Teacher, v101 n 6 p408-11 (Feb 2008)* that may help. It is available in the Evansdale Library.)

2. (a) Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be defined by $f(m, n) = 2m + n$. Is the function f an injection? Is the function f a surjection? Prove it.

(b) Let $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be defined by $g(m, n) = 6m + 3n$. Is the function g an injection? Is the function g a surjection? Prove it.

3. Prove that if A is countably infinite and B is finite, then $A \setminus B$ is countably infinite.

(Hint: Since $A \setminus B \subseteq A$, this means $A \setminus B$ is countable. Now assume $A \setminus B$ is finite and show that this leads to a contradiction.)

4. Prove that every subset of a countable set is countable.

(Hint: Let S be a countable set and assume that $A \subseteq S$. There are two cases: A is finite or A is infinite. If A is infinite, let $f : S \to \mathbb{N}$ be a bijection and define $g : A \to f(A)$ by $g(x) = f(x)$ for each $x \in A$.)