# Math 283 Spring 2012
# Assignment 2
# Solutions

**D'Angelo & West 2.2.**

*Solution.*

The statement is false, since assuming the two equations are true implies that a necessary condition for the existence of such integers $m$ and $n$ is that $(a+b)/2$ be an integer (i.e. $a+b$ be even). Thus, $(a, b) = (0, 1)$ is a counterexample.

Adding to the hypothesis the requirement that $a$ and $b$ have the same parity makes the statement true. In this case $m = (a+b)/2$ and $n = (a-b)/2$ are integers that satisfy the requirements for all such $a$ and $b$. ■

**D'Angelo & West 2.10.**

*Solution.*

For these solutions, we list the statement, a rephrasing as a conditional or a quantified statement, and the negation

(a) Every odd number is prime. Note that it is not relevant that this is false. If $x$ is an odd number, then $x$ is prime. Some odd number is composite (not prime).

(b) The sum of the angles of a triangle is 180 degrees. For every triangle $T$, the sum of the angles in $T$ is 180 degrees. Some triangle has an angle-sum greater than or less than 180 degrees (not 180 degrees).

(g) I get mad whenever you do that. If you do that, then I get mad. You do that without me getting mad. ■

**D'Angelo & West 2.38.**

Solution.

*Proof of (a).* Let $x$ and $y$ are odd. This means $x = 2k+1$ and $y = 2j+1$ for some integers $k$ and $j$. Consider $xy$.

$P \implies Q$

$$\begin{aligned} xy = (2k+1)(2j+1) &= 4kj + 2k + 2j + 1 \\ &= 2(kj + k + j) + 1 \end{aligned}$$

Since $kj + k + j$ is an integer, $2(kj + k + j) + 1$ is odd.

$Q \implies P$ via $\sim P \implies \sim Q$

Now, we need to prove if $x$ and $y$ are odd, then $xy$ is odd. We will proceed by proving the contrapositive statement, which is if $xy$ is not odd, then $x$ and $y$ are not both odd. If $x$ and $y$ are not both odd, then at least one is even. We can assume (by exchanging variables if necessary) that $x$ is the even integer. This means that $x = 2k$ for some integer $k$. Now, consider $xy = 2(ky)$, which (since $ky$ is an integer) is even. ■

*Part (b).* This statement is false. Let $x = 2k$ and $y = 2j + 1$, then $xy = 2(2jk + k)$, which (since $2jk + k$ is an integer) is even. Thus $xy$ is even, but $y$ is odd. ■

**D'Angelo & West 2.44.**

*Solution.*

(a) $(Q \wedge \sim Q) \implies P$. Recall that the conditional is false only when the hypothesis is true and the conclusion is false. For every statement $Q$, the hypothesis of this statement is false, and so the conditional is true regardless of whether $P$ is true.

(b) $P \wedge Q \implies P$. When $P$ and $Q$ are not both true, the hypothesis is false and so the conditional is true. When the hypothesis is true, we have that $P$ and $Q$ are both true and so the conclusion $P$ is true. Thus the conditional is always true.

(c) $P \implies P \vee Q$. When the hypothesis is true, $P$ is true, which means that $P$ or $Q$ is true regardless of the truth value of $Q$. Since the conclusion is true whenever the hypothesis is true, the conditional is always true. ■

**D'Angelo & West 2.48.**

*Solution.*

(a) This statement is false. Any odd integer suffices as a counterexample.

(b) This statement is true. The hypothesis is "All integers are odd", which is false, and so the conditional is true regardless of whether the conclusion ("All integers are even") is true or not. ■

**Extra Problem 1.**

*Solution.*

(a) The converse statement is "If $x^2 > 1$, then $x > 1$." The contrapositive is "If $x^2 \leq 1$, then $x \leq 1$." The negation is ""$x > 1$ and $x^2 \leq 1$".

(b) The converse is not true since $x = -2$ is a counterexample. The contrapositive is true since a contrapositive statement has the same truth value as the original statement. The negation is not true since for an $x \in \mathbb{R}$, $x^2 \leq 1$ means $|x| \leq 1$ which shows that $x$ cannot simultaneously be greater than 1. ■

**Extra Problem 2.**

*Solution.*

(a) False. A counterexample is $x = -1$.

(b) False. The only value that satisfies the inequality is $x = -1$ and $-1 \notin \mathbb{N}$.

(c) True. The value $x = -\frac{2W\left(\frac{\log(3)}{2}\right)}{\log(3)} \approx -.686027$ satisfies the equality.

(d) False. The value $x = -3$ is a counterexample.

(e) True. The value $x = 1$ works (43 is a prime number).

(f) False. The easiest one to find is when $x = 41$ which gives $41^2 + 41 + 41 = 1763$ which has a factor of 41 and so is not prime. The smallest counterexample is actually $x = 40$.

(g) False. Values that are very negative will show this is untrue. For example, when $x = -30$, the statement is that $-4560 \geq 0$ which is false. ■

**Extra Problem 3.**

*Solution.*

[By Contradiction] Suppose that $n$ is a natural number and $\frac{n}{n+1} \leq \frac{n}{n+2}$. Since $n$ is a natural number, $(n+1)(n+2)$ is positive and so we will preserve the inequality when multiplying both sides by $(n+1)(n+2)$. Doing so shows that $n(n+2) \leq n(n+1)$ which means $n^2 + 2n \leq n^2 + n$. Moving all of the terms to the left hand side, we have $n \leq 0$, but $n \in \mathbb{N}$. ↯.

Therefore $\frac{n}{n+1} > \frac{n}{n+2}$. ■

**Extra Problem 4.**

*Solution.*

(a)

| $(P$ | $\vee$ | $Q)$ | $\Longrightarrow$ | $(P$ | $\wedge$ | $Q)$ |
|---|---|---|---|---|---|---|
| T | T | T | **T** | T | T | T |
| T | T | F | **F** | T | F | F |
| F | T | T | **F** | F | F | T |
| F | F | F | **T** | F | F | F |

(b)

| $[(P$ | $\wedge$ | $Q)$ | $\vee$ | $(Q$ | $\wedge$ | $R)]$ | $\Longrightarrow$ | $(P$ | $\vee$ | $R)$ |
|---|---|---|---|---|---|---|---|---|---|---|
| T | T | T | **T** | T | T | T | **T** | T | **T** | T |
| T | T | T | **T** | T | F | F | **T** | T | **T** | F |
| T | F | F | **F** | F | F | T | **T** | T | **T** | T |
| T | F | F | **F** | F | F | F | **T** | T | **T** | F |
| F | F | T | **T** | T | T | T | **T** | F | **T** | T |
| F | F | T | **F** | T | F | F | **T** | F | **F** | F |
| F | F | F | **F** | F | F | T | **T** | F | **T** | T |
| F | F | F | **F** | F | F | F | **T** | F | **F** | F |

■