Math 283 Spring 2012
Assignment 1
Solutions

D'Angelo & West 1.1.

Solution.
The number of chairs (c) is at least ($\geq$) four times the number of tables (t), so $c \geq 4t$. ■

D'Angelo & West 1.10.

Solution.
The economy can absorb 100% PhD's, but 125% are being produced, so the unemployment would be $\frac{100}{125} = \frac{1}{5}$. ■

D'Angelo & West 1.13.

Solution.
Let $A = \{2k-1 : k \in \mathbb{Z}\}$ and $B = \{2k+1 : k \in \mathbb{Z}\}$. Let $n = 2k-1 \in A$ for some integer k. Notice that

$\boxed{A \subseteq B}$

$$\begin{aligned} n &= 2k-1 \\ &= (2k-2)+2-1 \\ &= 2(k-1)+1. \end{aligned}$$

Since $k-1 \in \mathbb{Z}$, we have $n \in B$. Thus, $A \subseteq B$.
Similarly, we let $m = 2j+1 \in B$ for some integer j. Notice that

$\boxed{B \subseteq A}$

$$\begin{aligned} m &= 2j+1 \\ &= (2j+2)-2+1 \\ &= 2(j+1)-1. \end{aligned}$$

Since $j+1 \in \mathbb{Z}$, we have $m \in A$. Thus, $B \subseteq A$.
Therefore, $A = B$. ■

D'Angelo & West 1.14.

Solution.
If $a < b < c < d$, then $[a,b] \cup [c,d]$ consists of all numbers in the closed interval $[a,d]$ except those between b and c. Thus $[a,b] \cup [c,d] = [a,d] \setminus (b-c)$. ■

D'Angelo & West 1.15.

Solution.

If $A = B$, then both differences are empty.

Method #1: Now, assume that $A \setminus B = B \setminus A$. Let $x \in A \setminus B$, this means $x \in A$ and $x \notin B$. However, $x \notin B$ means that $x \notin B \setminus A$. This means for equality to hold $A \setminus B$ and $B \setminus A$ must be empty.

Method #2: Consider the statement $A \setminus B = B \setminus A$. Set difference means that $A \setminus B$ contains no elements from B and that $B \setminus A$ contains no elements from A. Since they are equal, this must mean that they contain elements from neither A nor B and so each set difference must be empty. ■

D'Angelo & West 1.16.

Solution.

$5 \to 41 \to 32 \to 221 \to 311 \to 32$, reaching a cycle of length 3.

$6 \to 51 \to 42 \to 321 \to 321$, reaching a fixed point. ■

D'Angelo & West 1.18.

Solution.

If x is a real number such that it exceeds its reciprocal by 1, then $x = 1 + 1/x$. Since x cannot be 0, we can multiply by x, without changing the solutions, and obtain $x^2 - x - 1 = 0$. The solutions of this equation are $\dfrac{1 \pm \sqrt{5}}{2}$. ■

D'Angelo & West 1.23.

Solution.

This clock will be correct every 61 minutes, except between $12:12$ and $1:01$ between which there are only 49 minutes. ■

D'Angelo & West 1.24.

Solution.

There is no missing dollar, the correct accounting is $3 \cdot 9 - 2 = 25$, not $3 \cdot 9 + 2 \neq 30$. ■

D'Angelo & West 1.32.

Solution.

Let $S = \{x \in \mathbb{R} : x^2 - 2x - 3 < 0\}$ and $T = \{x \in \mathbb{R} : -1 < x < 3\}$.

$\boxed{T \subseteq S}$ ***Method #1:*** If $x \in T$, then $x + 1 > 0$ and $x - 3 < 0$. Hence $(x + 1)(x - 3) < 0$, which is the same as $x^2 - 2x - 3 < 0$. Thus $T \subseteq S$.

$\boxed{S \subseteq T}$ If $x \in S$, this means $x^2 - 2x - 3 < 0$, so $(x + 1)(x - 3) < 0$. The product of two numbers is negative only when exactly one of them is negative. This means $x < 3$ and $x > -1$. Thus $-1 < x < 3$ is the requirement and hence $S \subseteq T$.

Therefore, since $S \subseteq T$ and $T \subseteq S$, we have $S = T$.

Method #2: Since $x^2 - 2x - 3 = (x - 3)(x + 1)$ and the product of two numbers is negative precisely when exactly one of them is negative, S is the set of real numbers x such that exactly one of $x - 3$ and $x + 1$ is negative. Since $x - 3 < x + 1$, the negative one must be $x - 3$, and the condition is equivalent to $x - 3 < 0$ and $x + 1 > 0$. This becomes $x < 3$ and $x > -1$, which is the condition defining the set T. ■

Extra Problem 1.

Solution.

Many answers are possible.

(a) $A = \{x \in \mathbb{N} : 0 < x < 4\} = \{x \in \mathbb{Z} : 1 \leq x \leq 3\}$

(b) $B = \{x \in \mathbb{Z} : 0 \leq x \leq 3\}$

(c) $C = \{x \in \mathbb{R} : (x + 2)(x - 1) = 0\} = \{x \in \mathbb{N} : x - 3 < 0\}$ ■

Extra Problem 2.

Solution.

(a) $|A| = 5$

(b) $|B| = 11$

(c) $|C| = 2$

(d) $|D| = 1$ ■