1. Write an equation for the surface generated by revolving the curve $y^2 = 4x$ on the $xy$-plane around the $x$-axis.

**Solution** Consider a point $P(x, y, z)$ on the surface. Then there is a point $Q(x, y_1, 0)$ on the curve $y^2 = 4x$ rotating about the $x$-axis to get $P$. Both $P$ and $Q$ must have the same distance to the $x$-axis, or more precisely, to the point $C(x, 0, 0)$ on the $x$-axis. As $|PC| = |QC|$, we have

$$(x - x)^2 + (y - 0)^2 + (z - 0)^2 = (x - x)^2 + (y_1 - 0)^2 + (0 - 0)^2,$$

and so $y_1^2 = y^2 + z^2$. As $(x, y_1)$ is a point on the curve, we also have $y_1^2 = 4x$. Combine these equations to get the answer

$$y^2 + z^2 = 4x.$$

2. Convert the equation $x^2 + y^2 + z^2 = x + y + z$ to both cylindrical and spherical coordinates.

**Solution** For cylindrical coordinates, apply the conversion formulas to get

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta + z^2 = r \cos \theta + r \sin \theta + z.$$

For spherical coordinates, apply the conversion formulas to get

$$\rho^2 = \rho \sin \phi \cos \theta + \rho \sin \phi \sin \theta + \rho \cos \phi \text{ or } \rho = \sin \phi \cos \theta + \sin \phi \sin \theta + \cos \phi.$$