Review for Exam 3

1. How to find tangent planes at a point \( P(a, b, c) \)? (For a surface with equation \( z = f(x, y) \), use normal vector \( \vec{n}(a, b) = \langle f_x(a, b), f_y(a, b), -1 \rangle \). For surface with equation \( F(x, y, z) = 0 \), the gradient \( \nabla F(a, b, c) \) is a normal.

2. How to classify the critical points (if they are local extrema and what kind)? (Use \( \Delta(x, y) = f_{xx}f_{yy} - (f_{xy})^2 \).)

3. Differentials and its applications.
\[
\text{df}(x, y, z) = \nabla f(x, y, z) \cdot < dx, dy, dz > .
\]
One can use \( df \) as an approximation to \( f(x + dx, y + dy, z + dz) - f(x, y, z) \).

4. Chain rules: (Go check them on pages 736 and 739).

5. Implicit partial differentiations. An equation \( F(x, y, z) = \text{constant} \), defines one variable (\( z \), say) as a function of the other variables (\( x, y \), say). Then
\[
\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}.
\]

6. The gradient and the directional derivatives:
\[
\nabla f(x, y, z) = \langle f_x, f_y, f_z \rangle \quad \text{and} \quad D_\vec{n}f(x, y, z) = \nabla f(x, y, z) \cdot \vec{n}, \quad \text{if } |\vec{n}| = 1.
\]

7. Evaluation of double integrals: (\( x, y \) coordinates)
\[
\int \int_R f(x, y) dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) dy dx, \quad \text{if } R \text{ is } a \leq x \leq b, \quad g_1(x) \leq y \leq g_2(x).
\]
\[
\int \int_R f(x, y) dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) dy dx, \quad \text{if } R \text{ is } c \leq y \leq d, \quad h_1(y) \leq x \leq h_2(y).
\]
When you can evaluate the integral by either way, you may want to choose a simpler way.

8. Evaluation of double integrals: (Polar coordinates)
\[
\int \int_R f(x, y) dA = \int_a^b \int_{g_1(r)}^{g_2(r)} f(r \cos \theta, r \sin \theta) r dr d\theta, \quad \text{if } R \text{ is } a \leq r \leq b, \quad g_1(r) \leq \theta \leq g_2(r).
\]
\[
\int \int_R f(x, y) dA = \int_a^b \int_{h_1(\theta)}^{h_2(\theta)} f(r \cos \theta, r \sin \theta) r dr d\theta, \quad \text{if } R \text{ is } a \leq \theta \leq b, \quad h_1(\theta) \leq r \leq h_2(\theta).
\]
A useful fact: \( dA = rdrd\theta = dx dy \).

9. Some applications of double integrals.

9a. Area of region \( R \) is \( \int \int_R dA \).

9b. The volume between \( z = f(x, y) \) and \( z = g(x, y) \) when \( (x, y) \) are in \( R \) is \( \int \int_R (f(x, y) - g(x, y)) dA \).