1. Write out all the terms of $\sum_{i=3}^{7}(i^2 + 1)$ and compute the sum.

2. Use summation rules to compute the sum $\sum_{i=2}^{70}(i^2 + 1)$.

3. Use summation rules to compute the sum $\sum_{i=2}^{20}(i^2 + i + 1)(i - 1)$.

4. Compute the sum $f(n) = \sum_{i=1}^{n} \frac{1}{n} \left[ \left( \frac{i}{n} \right)^2 - 3 \left( \frac{i}{n} \right) + 1 \right]$, and compute the limit $\lim_{n \to \infty} f(n)$. 


5. Let \( n > 0 \) be an integer. Suppose that \( f(x) \) is an increasing function on the interval \([a, b]\). Let \( R'_n, R''_n \) be the Riemann sums of \( f(x) \) on \([a, b]\) for the regular partition of \([a, b]\) into \( n \) equal length subintervals, using the left point evaluation and using the right point evaluation, respectively. Which of the following is correct: \( R'_n \geq R''_n \)? \( R''_n > R'_n \)? or We cannot conclude either of these two inequalities? Explain your answer.

6. Approximate the area under the curve \( y = x^2 + 1 \) and above the \( x \)-axis on the interval \([0, 2]\) by computing the Riemann sum with \( n = 4 \) equal length subintervals of \([0, 2]\) and by using the right point evaluation.

7. Compute the Riemann sum of \( f(x) = x^2 - 1 \) for the regular partition of the interval \([0, 2]\) into \( n = 100 \) equal length subintervals, using the right point evaluation.
1. Write out all the terms of $\sum_{i=3}^{7}(i^2 + 1)$ and compute the sum.

Solution: Compute

$$\sum_{i=3}^{7}(i^2 + 1) = (9 + 1) + (16 + 1) + (25 + 1) + (36 + 1) + (49 + 1) = 10 + 17 + 26 + 37 + 50 = 140.$$ 

2. Use summation rules to compute the sum $\sum_{i=2}^{70}(i^2 + 1)$.

Solution: Compute

$$\sum_{i=2}^{70}(i^2 + 1) = \sum_{i=1}^{70}(i^2 + 1) - (1^2 + 1) = \sum_{i=1}^{70}i^2 + \sum_{i=1}^{70}1 - 2 = \frac{70(70 + 1)(140 + 1)}{6} + 70 - 2 = 35 \cdot 71 \cdot 47 + 68 = 116795 + 68 = 116863.$$ 

3. Use summation rules to compute the sum $\sum_{i=2}^{70}(i^2 + i + 1)(i - 1)$.

Solution: First compute $(i^2 + i + 1)(i - 1) = i^3 - 1$. Thus

$$\sum_{i=2}^{20}(i^2 + i + 1)(i - 1) = \sum_{i=2}^{20}(i^3 - 1) = \sum_{i=1}^{20}i^3 - \sum_{i=1}^{20}1 = \frac{20^2(20 + 1)^2}{4} - 20 = 100 \cdot 441 - 20 = 44080.$$

4. Compute the sum $f(n) = \sum_{i=1}^{n} \frac{1}{n} \left[ \left( \frac{i}{n} \right)^2 - 3\left( \frac{i}{n} \right) + 1 \right]$, and compute the limit $\lim_{n \to \infty} f(n)$.

Solution: First compute

$$f(n) = \sum_{i=1}^{n} \frac{1}{n} \left[ \left( \frac{i}{n} \right)^2 - 3\left( \frac{i}{n} \right) + 1 \right]$$

$$= \frac{1}{n} \sum_{i=1}^{n} \left[ \left( \frac{i}{n} \right)^2 - 3 \frac{i}{n} + 1 \right]$$

$$= \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - 3 \frac{1}{n^2} \cdot \frac{n(n+1)}{2} = \frac{(n+1)(2n+1)}{6n^2} - 3 \frac{(n+1)}{2n} + 1$$

Therefore,

$$\lim_{n \to \infty} f(n) = \lim_{n \to \infty} \frac{(n+1)(2n+1)}{6n^2} - \frac{3(n+1)}{2n} + 1 = \frac{1}{3} - \frac{3}{2} + 1 = -\frac{1}{6}.$$
5. Let \( n > 0 \) be an integer. Suppose that \( f(x) \) is an increasing function on the interval \([a, b]\). Let \( R'_n, R''_n \) be the Riemann sums of \( f(x) \) on \([a, b]\) for the regular partition of \([a, b]\) into \( n \) equal length subintervals, using the left point evaluation and using the right point evaluation, respectively. Which of the following is correct: \( R'_n \geq R''_n \)? \( R''_n > R'_n \)? or We cannot conclude either of these two inequalities? Explain your answer.

**Solution:** Let \( x_0, x_1, \ldots, x_n \) denote the regular partition points on \([a, b]\). Then in the \( i \)th subinterval \([x_{i-1}, x_i]\), the left end is \( x_{i-1} \) and the right end if \( x_i \). Since \( f(x) \) is increasing, \( f(x_{i-1}) \leq f(x_i) \), and so

\[
R'_n = \sum_{n=1}^{n} f(x_{i-1}) \Delta x \leq \sum_{n=1}^{n} f(x_i) \Delta x = \Delta x = R''_n.
\]

6. Approximate the area under the curve \( y = x^2 + 1 \) and above the \( x \)-axis on the interval \([0, 2]\) by computing the Riemann sum with \( n = 4 \) equal length subintervals of \([0, 2]\) and by using the right point evaluation.

**Solution:** Here \( a = 0, b = 2 \) and \( n = 4 \). Thus

\[
\Delta x = \frac{b - a}{n} = \frac{2}{4} = \frac{1}{2}, x_i = a + i \Delta = \frac{i}{2}
\]

Let \( f(x) = x^2 + 1 \). Then the Riemann sum is

\[
R_4 = \frac{1}{2} \sum_{i=1}^{4} f(x_i) = \frac{1}{2} \sum_{i=1}^{4} \left( \frac{i}{2} \right)^2 + 1 = \frac{1}{2} \left[ \left( \frac{1}{4} + 1 \right) + \left( \frac{4}{4} + 1 \right) + \left( \frac{9}{4} + 1 \right) + \left( \frac{16}{4} + 1 \right) \right] = \frac{23}{4}.
\]

7. Compute the Riemann sum of \( f(x) = x^2 - 1 \) for the regular partition of the interval \([0, 2]\) into \( n = 100 \) equal length subintervals, using the right point evaluation.

**Solution:** Here \( a = 0, b = 2 \) and \( n = 100 \). Thus

\[
\Delta x = \frac{b - a}{n} = \frac{2}{100} = \frac{1}{50}, x_i = a + i \Delta = \frac{i}{50}
\]

Let \( f(x) = x^3 - 1 \). Then the Riemann sum is

\[
R_{100} = \frac{1}{50} \sum_{i=1}^{100} f(x_i) = \frac{1}{50} \sum_{i=1}^{100} \left[ \left( \frac{i}{50} \right)^2 \right] = \frac{1}{50^3} \left[ \sum_{i=1}^{100} i^2 - 100 \right] = \frac{1}{125000} \cdot [100(100 + 1)/2 - 100] = \frac{1}{125000} \cdot 4950 = \frac{99}{2500}.
\]