On Perfect Matching Coverings and Even Subgraph Coverings

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Abstract: A perfect matching covering of a graph G is a set of perfect matchings of G such that every edge of G is contained in at least one member of it. Berge conjectured that every bridgeless cubic graph admits a perfect matching covering of order at most 5 (we call such a collection of perfect matchings a Berge covering of G). A cubic graph G is called a Kotzig graph if G has a 3-edge-coloring such that each pair of colors forms a hamiltonian circuit (introduced by R. Häggkvist, K. Markström, J Combin Theory Ser B 96 (2006), 183–206). In this article, we prove that if there is a vertex w of a cubic graph G such that $\overline{G-w}$, the graph obtained from $G-w$ by suppressing all degree two vertices is a Kotzig graph, then G has

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a Berge covering. We also obtain some results concerning the so-called 5-even subgraph double cover conjecture. J. Graph Theory 81: 83–91, 2016

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1. INTRODUCTION

Let G be a graph that may contain multiple edges. A matching M is a 1-regular subgraph of G. A perfect matching of G is a spanning 1-regular subgraph of G (also called a 1-factor of G), and an r-factor of G is a spanning r-regular subgraph of G. A circuit is a connected 2-regular graph, and an even subgraph (also called a cycle) is a subgraph such that each vertex has an even degree. Let G be a graph. The suppressed graph, denoted by $\overline{G}$, is the graph obtained from G by suppressing all degree two vertices.

Definition 1.1. *Let G be a bridgeless cubic graph.*

(i) *A perfect matching covering $\mathcal{M}$ of G is a multiset of perfect matchings of G if every edge of G is contained in at least one member of $\mathcal{M}$. Let $\mathcal{T}_\mu$ be the set of cubic graphs admitting perfect matching coverings $\mathcal{M}$ with $|\mathcal{M}| = \mu$.*
(ii) *A perfect matching covering $\mathcal{M}$ of G is a $(1,2)$-covering if every edge of G is contained in precisely one or two members of $\mathcal{M}$. Let $\mathcal{T}^\star_\mu$ be the set of cubic graphs admitting perfect matching $(1,2)$-coverings $\mathcal{M}$ with $|\mathcal{M}| = \mu$.*

A. Matching Coverings

The following conjecture appears first in [6], and is attributed to Berge in [16].

Conjecture 1.2. *(Berge and Fulkerson) Every bridgeless cubic graph G is in $\mathcal{T}^\star_6$.*

We call such a perfect matching covering in Conjecture 1.2*a Fulkerson covering*. If a bridgeless cubic graph G has a Fulkerson covering, then any five members of a Fulkerson covering of G can cover the edge set of G. This naturally yields a weaker version of Conjecture 1.2 also attributed to Berge.

Conjecture 1.3. *(Berge) Every bridgeless cubic graph G is in $\mathcal{T}_5$.*

The perfect matching covering in Conjecture 1.3 is called a *Berge covering* of G.

Let $\mathcal{G}$ be the set of all bridgeless cubic graphs. The following are some earlier results and some straightforward observations for *cubic* graphs.

Observation 1.4. *For cubic graphs,*

$$\mathcal{T}^\star_3 = \mathcal{T}_3 \subset \left\{ \begin{array}{ccc} \mathcal{T}^\star_4 & = & \mathcal{T}_4 \quad \subset \\ \mathcal{T}^\star_5 & = & \mathcal{T}^\star_6 \quad \subseteq \end{array} \right\} \mathcal{T}_5 \subseteq \mathcal{T}_6 \subseteq \mathcal{G}.$$

But it remains *unknown* whether $\mathcal{T}^\star_4 \subseteq \mathcal{T}^\star_5$ (see Conjecture 4.9).

Observation 1.5. *$\mathcal{T}_3$ is the set of all 3-edge-colorable cubic graphs.*

For cubic graphs,

$$\mathcal{T}^\star_6 \subseteq \mathcal{T}_5.$$

However, it remains *unknown* whether $\mathcal{T}_5 = \mathcal{T}_6^\star$. Under the assumption that $\mathcal{T}_5 = \mathcal{G}$, Mazzuoccolo [14] proved the following theorem.

Theorem 1.6. *[14] If $\mathcal{T}_5 = \mathcal{G}$, then $\mathcal{T}_5 = \mathcal{T}_6^\star$.*

Mazzuoccolo [14] proved that Conjecture 1.2 and Conjecture 1.3 are equivalent. However, the equivalency of Conjecture 1.2 and Conjecture 1.3 remains unknown for a given graph. That motivates our investigation to the size of perfect matching coverings of a given cubic graph.

Hao et al. [7] gave a sufficient and necessary condition for a given cubic graph $G \in \mathcal{T}_6^\star$. They proved the following lemma.

Lemma 1.7. *[7] Given a cubic graph G, $G \in \mathcal{T}_6^\star$ if and only if there are two edge-disjoint matchings M_1 and M_2 such that each suppressed graph $\overline{G \setminus M_i}$ is 3-edge-colorable for $i = 1, 2$ and $M_1 \cup M_2$ forms an even subgraph in G.*

B. A Family of Berge Coverable Graphs

We call a graph G *hypohamiltonian* if G itself is not hamiltonian but $G - v$ has a hamiltonian circuit for any vertex v of G [9]. A snark is a non-3-edge-colorable cubic graph. Conjectures 1.2 and 1.3 are trivial for 3-edge-colorable cubic graphs, especially for hamiltonian cubic graphs.

A cubic graph G is called *a Kotzig graph* if G is 3-edge-colorable such that each pair of colors forms a hamiltonian circuit (defined by Häggkvist and Markström in [8]). A cubic graph G is called *an almost Kotzig graph* if, there is a vertex w of G, such that the suppressed graph $\overline{G - w}$ is a Kotzig graph. In this article, we prove that Berge's conjecture holds for almost Kotzig graphs G (Theorem 2.1).

C. Even Subgraph Covering

Let G be a graph. An *even subgraph covering* of a graph G is a family of even subgraphs $\mathcal{C}$ of G with the property that each edge of G is contained in at least one member of $\mathcal{C}$. An even subgraph covering $\mathcal{C}$ of G is called an even subgraph double cover if each edge of G is contained in precisely two members of $\mathcal{C}$. The even subgraph (or cycle/circuit) double cover conjecture is one of the major open problems in graph theory (see [12, 17, 20], or see [22]). The following stronger version of the circuit double cover conjecture was proposed by Celmins and Preissmann.

Conjecture 1.8. *(Celmins [2] and Preissmann [15]) Every bridgeless graph G has a* 5-*even subgraph double cover.*

Note that, by applying Fleischner's vertex-splitting lemma [3], it suffices to prove Conjecture 1.8 for onlycubic graphs.

$\mathcal{T}_4$-graphs (the family of cubic graphs that are covered by a set of four perfect matchings) is a very special family, which contains all 3-edge-colorable cubic graphs, and some snarks, for example, the flower snark, the Goldberg snark, and the permutation snark except for the Petersen graph [5]. In this article, we discover some closed relation between 5-even subgraph double cover conjecture and $\mathcal{T}_4$-property (Theorems 3.5 and 3.3).

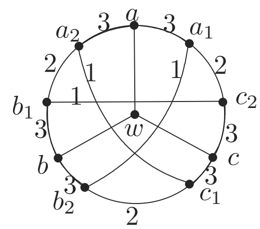
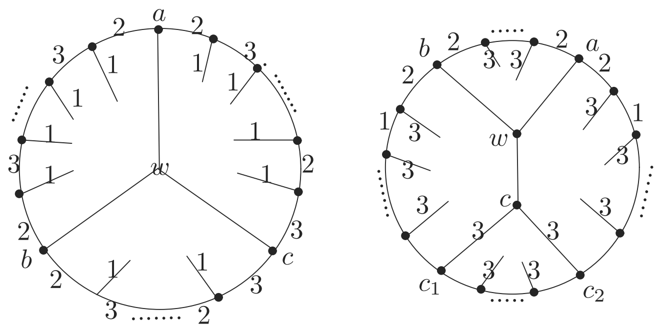


FIGURE 1. The coloring of the Petersen graph.



(a) (b)

FIGURE 2. Coloring of Almost Kotzig graph.

2. ALMOST KOTZIG GRAPHS

The following theorem verifies Berge's conjecture for every almost Kotzig graph.

Theorem 2.1. *Let G be an almost Kotzig graph. Then $G \in \mathcal{T}_5$. That is, every almost Kotzig graph has a Berge covering.*

Proof. We need only prove that G admits a Fulkerson covering or a Berge covering. Let $n = |V(G)|$. Then n is even. Since $\overline{G-w}$ is Kotzigian, $\overline{G-w}$ has an edge coloring $f : E(\overline{G-w}) \to \{1, 2, 3\}$ such that each pair of colors forms a hamiltonian circuit of $\overline{G-w}$. Let $N_G(w) = \{a, b, c\}$. For $x \in \{a, b, c\}$, let x_1 and x_2 denote the neighbors of x other than w. We sketch the proof of each case.

case 1. a_1a_2, b_1b_2 and c_1c_2 have the same color, say 3.
In this case, the edges of colors 1 and 2 form a circuit of G consisting of two matchings, say M_1 and M_2 (see Fig. 1). By the Kotzigian of $\overline{G-w}$, $\overline{G \setminus M_i} \cong K_4$ is 3-edge-colorable. The result follows from Lemma 1.7.

case 2. a_1a_2, b_1b_2, and c_1c_2 are assigned two colors.
Without loss of generality, assume a_1a_2 and b_1b_2 have color 2 and c_1c_2 has color 3. By the Kotzigian of $\overline{G-w}$, the edges of colors 2 and 3 form a circuit of length $n-1$ in G (see Fig. 2 a). Then we can construct two perfect matchings covering the edges wa, wb and the edges of color 3 and some of the edges of color 2.
Again by $\overline{G-w}$ is Kotzigian, the edges of colors 1 and 2 form a circuit C' of length $n-2$ (it is even) in G (see Fig. 2 b). Partition C' into two matchings and add wc to them, respectively. We get two perfect matchings of G covering

the edges wc and the edges of colors 1 and 2. Therefore, we construct a Berge covering of G.

case 3. a_1a_2, b_1b_2, and c_1c_2 have pairwise different colors.

In this case, the edges of two different colors form a circuit of length $n-2$ in G by the Kotzigian of $\overline{G-w}$. Then with the same construction as in the second paragraph of Case 2 and the symmetry of the colors, we can construct six perfect matchings of G covering each edge of G precisely twice. This completes the proof. ■

Remark: As we have shown in the proof of Theorem 2.1, the circumference of an almost Kotzig graph on n vertices is $n-1$ or $n-2$. Discovered by Brinkmann et al. [1], the number of snarks of order n with circumference $n-1$ or $n-2$ has a sudden increase as $n \geq 34$. We believe that Theorem 2.1 extends the family of snarks admitting a Fulkerson covering or a Berge covering, but it is not an easy task to find a snark that is almost Kotzig but does not belong to the well-known families of snarks admitting Berge–Fulkerson conjecture, such as the flower snarks, the Goldberg snarks, the generalized Blanusa snarks, and Loupekine snarks [4, 7, 13].

3. MATCHING COVERINGS AND CYCLE COVERINGS

$\mathcal{T}_4$-graphs are somehow special, and are expected to have some special graph theory properties. As we mentioned above, the Petersen graph P_{10} is not a $\mathcal{T}_4$-graph (Proposition 4.7).

Conjecture 1.8 has been verified by Huck and Kochol [11] and Huck [10] for oddness 2 and oddness 4 graphs. Here, we will verify Conjecture 1.8 for graphs in $\mathcal{T}_4$.

Theorem 3.1. *If $G \in \mathcal{T}_4$, then G has a 5-even subgraph double cover.*

Theorem 3.1 was also proved by Steffen 2012 independently. In the following, we present two stronger versions.

Let G be a graph. A subgraph P of G is a parity subgraph of G if $d_P(v) \equiv d_G(v) \pmod 2$ for every vertex v of G. It is evident that P is a parity subgraph if and only if $G - E(P)$ is even.

Note that, graphs considered here may not necessarily be cubic.

Definition 3.2. *Let G be a graph. A set $\mathcal{P}$ of parity subgraphs of G is a $(1,2)$-covering if every edge of G is contained in precisely one or two members of $\mathcal{P}$. Let $\mathcal{S}^\star_\mu$ be the set of graphs admitting parity subgraph $(1,2)$-coverings $\mathcal{P}$ with $|\mathcal{P}| = \mu$.*

Clearly, $\mathcal{T}_4 \subseteq \mathcal{S}^*_4$. The following is a stronger version of Theorem 3.1. It is also an equivalent version for the 5-even subgraph double cover problem.

Theorem 3.3. *Let G be a graph. The following statements are equivalent.*

(1) *$G \in \mathcal{S}^\star_4$;*
(2) *G has a 5-even subgraph double cover.*

In the proof of Theorem 3.3, we will use the following observation.

Observation 3.4. *Let $\{P_1, \ldots, P_t\}$ be a set of parity subgraphs of a graph G. Let $\triangle_{i=1}^t P_i$ be the symmetric difference of $P_1, \ldots, P_t$. Then $\triangle_{i=1}^t P_i$ is a parity subgraph (or even subgraph) of G if t is odd (or, even, respectively).*

Proof of Theorem 3.3. (1) $\Rightarrow$ (2): Let $\mathcal{P} = \{P_1, \ldots, P_4\}$ be a set of parity subgraph (1, 2)-covering of G. Denote

$$E_\mu = \{e \in E(G) : \ e \text{ is covered by } \mathcal{P} \ \mu\text{-times}\}.$$

By Observation 3.4, $E_1 = \triangle_{i=1}^4 P_i$ is an even subgraph of G. And $Q_i = G - E(P_i)$ is also an even subgraph ($i = 1, 2, 3, 4$).

Note that $\{E_1 \triangle Q_i : i = 1, \ldots, 4\}$ covers E_1 once, E_2 twice. Thus

$$\{E_1 \triangle Q_i : i = 1, \ldots, 4\} \cup \{E_1\}$$

covers $E(G)$ $(= E_2 \cup E_1)$ precisely twice. Hence, it is a 5-even subgraph double cover of G.

(2) $\Rightarrow$ (1): Let $\{C_0, C_1, \cdots, C_4\}$ be a 5-even subgraph double cover of G. Then we can see that $\{C_0 \triangle C_i : i = 1, 2, 3, 4\}$ covers every edge twice or three times, since, for each edge $e \in E(G)$, $\{C_0 \triangle C_i : i = 1, 2, 3, 4\}$ covers e twice if $e \notin E(C_0)$, or three times if $e \in E(C_0)$. So, $P_i = G - E(C_0 \triangle C_i)$ is a parity subgraph, and $\{P_1, \ldots, P_4\}$ covers each edge e once if $e \in E(C_0)$, or, twice if $e \notin E(C_0)$. ■

The following is an analogy of Theorem 3.3 for perfect matching covering of cubic graphs, and is another stronger version of Theorem 3.1.

Theorem 3.5. *Let G be a cubic graph. The following statements are equivalent.*

(1) *$G \in \mathcal{T}_4$;*
(2) *G has a 5-even subgraph double cover $\{C_0, \ldots, C_4\}$ with C_0 as a 2-factor of G.*

Proof. We just note that perfect matchings of cubic graphs G are parity subgraphs of G and the symmetric difference of a covering of four perfect matchings is a 2-factor of G. The rest of the proof is the same as Theorem 3.3. ■

4. CONCLUDING REMARKS

In this section, we give some results and problems concerning perfect matching covering, parity subgraph covering, and even subgraph covering.

Definition 4.1. *Let G be a cubic graph. Let F be a spanning even subgraph of G. The oddness of F is the number of odd components of F. The oddness of the graph G is the minimum oddness of all spanning even subgraphs F.*

Note that the spanning even subgraph in the above definition may not be a 2-factor (see [22]). It is proved in [10] that every bridgeless graph with oddness at most 4 has a 5-even subgraph double cover. Hence, by Theorem 3.3, we have the following theorem.

Theorem 4.2. *Every bridgeless cubic graph with oddness at most 4 is a $\mathcal{S}_4^\star$-graph.*

By Theorem 4.2, we have the following corollaries for some oddness 2 graphs.

Corollary 4.3.

(i) *Every permutation graph is a $\mathcal{S}_4^\star$-graph;*
(ii) *Every hypohamiltonian graph is a $\mathcal{S}_4^\star$-graph;*
(iii) *Every almost Kotzig graph is a $\mathcal{S}_4^\star$-graph.*

Proof. It is easy to see that every permutation graph and every hypohamiltonian graph is of oddness at most 2.

Let G be an almost Kotzig graph. Since $\overline{G-w}$ is Kotzigian for some vertex w, $\overline{G-w}$ has an edge coloring $f : E(\overline{G-w}) \rightarrow \{1, 2, 3\}$ such that each pair of colors forms a hamiltonian circuit of $\overline{G-w}$. Let $N_G(w) = \{a, b, c\}$. For $x \in \{a, b, c\}$, let x_1 and x_2 denote the neighbors of x other than w.

If edges a_1a_2, b_1b_2, and c_1c_2 are colored by at most two colors, say $\{1, 2\}$, then the subgraph induced by 1 and 2 colored edges, plus the isolated vertex w is spanning and even, and, therefore, G is of oddness at most 2.

If edges a_1a_2, b_1b_2, and c_1c_2 are colored by all three colors, say $f(c_1c_2) = 3$, then the subgraph induced by 1 and 2 colored edges, plus two isolated vertices (not an edge) $\{w, c\}$ is spanning and even, and, therefore, G is of oddness at most 2. ■

Due to the structural difference between perfect matchings and parity subgraphs, it remains unknown whether a $\mathcal{S}_4^\star$-graph is a $\mathcal{T}_4$- or $\mathcal{T}_5$-graph.

Problem 4.4. *For every given cubic graph G, is it true that if $G \in \mathcal{S}_4^\star$ then $G \in \mathcal{T}_4$ or $G \in \mathcal{T}_5$?*

That is, can we prove Berge Conjecture (Conjecture 1.3) for $\mathcal{S}_4^\star$-graphs? It is not hard to obtain inclusion relations similar to Observation 1.4 for parity subgraph covering. The following result(similar to Observation 1.5) is for $\mathcal{S}_3^\star$-graphs.

Theorem 4.5. *Let G be a graph. The following statements are equivalent.*

(i) *$G \in \mathcal{S}_3^\star$;*
(ii) *G has a parity subgraph decomposition $\{P_1, P_2, P_3\}$;*
(iii) *G has a* 3*-even subgraph double cover;*
(iv) *G admits a nowhere-zero* 4*-flow.*

Proof. (ii) ⇒ (i) is trivial.

(i) ⇒ (ii): Let $\mathcal{P} = \{P_1, P_2, P_3\}$ be a parity subgraph $(1, 2)$-covering of G.

$$\{G - (P_i \triangle P_j) : 1 \leq i < j \leq 3\}$$

is a parity subgraph decomposition of G.

The equivalence of (ii), (iii), and (iv) can be found in [22]. ■

Although perfect matching covering and parity subgraph covering are different in structure, for cubic graphs, $\mathcal{T}_3$-graphs and $\mathcal{S}_3^\star$-graphs are the same.

Corollary 4.6. *For cubic graphs, $\mathcal{T}_3 = \mathcal{S}_3^\star$. That is, a cubic $\mathcal{S}_3^\star$-graph is* 3*-edge-colorable.*

Although every cubic graph is conjectured to have a 5-even subgraph double cover (Conjecture 1.8), not every graph has the $\mathcal{T}_4$-property. The Petersen graph is an example. The Petersen graph has precisely six perfect matchings, and each pair of them intersects at precisely one edge. Thus, we have the following proposition.

Proposition 4.7. *(Fouquet and Vanherpe, Theorem 3.12 in [5]) The Petersen graph P_{10} is not a member of $\mathcal{T}_4$.*

We propose the following conjectures.

Conjecture 4.8. *For every bridgeless cubic graph G, if G is Petersen-minor-free, then* $G \in \mathcal{T}_4$.

This is a weak version of a conjecture by Tutte that $G \in \mathcal{T}_3$ for every bridgeless Petersen-minor-free cubic graph G. Although the proof of this Tutte's conjecture was announced by Robertson, Sanders, Seymour, and Thomas ([18, 21]), a simplified manual proof of Conjecture 4.8 will certainly develop some new techniques in graph theory, and, therefore, it remains as an interesting research problem in graph theory.

Conjecture 4.9. *For every given cubic graph G, if* $G \in \mathcal{T}_4$ *then* $G \in \mathcal{T}_6^\star$. *That is, Berge–Fulkerson conjecture is true for all graphs of* $\mathcal{T}_4$.

Conjecture 4.10. *For every given snark G,* $G \in \mathcal{T}_5$ *if and only if* $G \in \mathcal{T}_6^\star$.

Conjecture 4.10 implies the equivalence of Conjecture 1.2 and Conjecture 1.3 for every given graph (a further improvement of Theorem 1.6). Some families of cubic graphs have been confirmed as $\mathcal{T}_5$-graphs (e.g., permutation graphs ([5], Theorem 3.12), almost Kotzig graphs (Theorem 2.1), etc.). The verification of Conjecture 4.10 will further extend those results for Berge–Fulkerson conjecture.

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