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Uniquely forced perfect matching and unique 3-edge-coloring

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ABSTRACT

Let G be a cubic graph with a perfect matching. An edge e of G is a *forcing edge* if it is contained in a unique perfect matching M, and the perfect matching M is called *uniquely forced*. In this paper, we show that a 3-connected cubic graph with a uniquely forced perfect matching is generated from K_4 via $Y \to \Delta$-operations, i.e., replacing a vertex by a triangle, and further a cubic graph with a uniquely forced perfect matching is 2-connected and contains a triangle. Our result generalizes a previous result of Jiang and Zhang (2011). The unique 3-edge-coloring conjecture asserts that a Petersen-minor-free cubic graph with a unique 3-edge-coloring must contain a triangle. Our result verifies that the unique 3-edge-coloring conjecture holds for a subfamily of uniquely 3-edge-colorable cubic graphs, namely cubic graphs with uniquely forced perfect matching.

1. Introduction

In the paper, graphs may not contain multiple edges and loops, otherwise multi-graphs are used instead. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. A subset M of $E(G)$ is a *matching* of G if any two edges in M are disjoint. A matching M is *perfect* if every vertex of G is incident with exactly one edge in M. In theoretical chemistry, perfect matchings of a graph (modeling a molecule) have physical meanings and are called *Kekulé structures* of the molecule. A subset S of a perfect matching M is called a *forcing set* if M is the unique perfect matching containing S. The *forcing number* (or degree of freedom in theoretical chemistry, [9]) of M is the minimum cardinality of forcing sets of M, denoted by $f(G; M)$. If S contains only one edge e, then M is called *uniquely forced* and the edge e is called a *forcing edge* of G. The *forcing number* of a graph G is the minimum number of forcing numbers of all perfect matchings of G, denoted by $f(G)$. So a graph G with a forcing edge has $f(G) = 1$. It is NP-complete [1] to determine the forcing set of a perfect matching of a bipartite graph with maximum degree 3.

The forcing number of a molecular graph (the skeleton of a molecule) has applications in theoretical chemistry [9,11]. It has been revealed that a stable molecule has relatively large forcing number [11]. In this sense, a molecule with a forcing edge is unstable. Benzenoid hydrocarbons with forcing edges have been investigated by Zhang and Li [21,22]. Later, plane bipartite graphs with forcing edges have been studied in [24]. The forcing number problem have been well-studied for many chemical graphs, including fullerenes [23] and BN-fullerenes [10], some important families of plane cubic graphs. The following is a recent result obtained by Jiang and Zhang [10].

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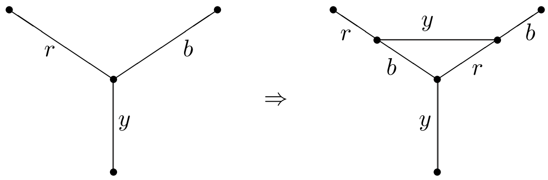

Fig. 1. $Y \to \Delta$-operation.

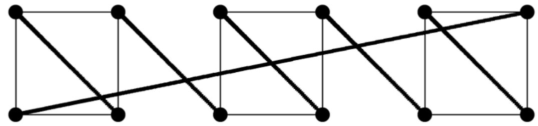

Fig. 2. A non-3-edge-connected cubic graph with a forcing edge which is not uniquely 3-edge-colorable.

Theorem 1.1 (*[10]*). *Let G be a plane cubic graph such that every face is bounded by a quadrangle or a hexagon. Then the forcing number of G is at least* 2*.*

In this paper, we investigate the structure properties of cubic graphs with a uniquely forced perfect matching. Let G be a cubic graph. A $Y \to \Delta$*-operation* on a vertex v of G means replacing v by a triangle (see Fig. 1). The following is our main result.

Theorem 1.2. *Let G be a* 3*-connected cubic graph. If G has a forcing edge, then G is generated from K_4 via a series of $Y \to \Delta$-operations.*

A more general result can be derived from the above theorem as follows.

Theorem 1.3. *Every cubic graph with a forcing edge is* 2*-connected and contains a triangle.*

As a direct corollary of Theorem 1.3, the following result generalizes Theorem 1.1 to triangle-free cubic graphs.

Corollary 1.4. *Let G be a triangle-free cubic graph. Then the forcing number of G is at least* 2*.*

The above results show that a stable molecule unlikely contains a triangle which is usually considered as a unstable structure in theoretical chemistry. Besides this, Theorem 1.2 has connections to the unique 3-edge-coloring problem of cubic graphs.

A cubic graph G is 3*-edge-colorable* if the edges of G can be colored by three colors such that any two adjacent edges have different color. In other words, the edges of G can be partitioned to three disjoint perfect matchings, also called 1*-factorization*. A cubic graph G is *uniquely* 3*-edge-colorable* if it has a unique 3-edge-coloring.

Conjecture 1.5 (*Fiorini–Wilson–Fisk, [5,6]*). *Let G be a planar cubic graph. If G is unique* 3*-edge-colorable, then G contains a triangle.*

Conjecture 1.5 is verified by Fowler and Thomas [7] by using a method similar to the proof of the 4-Color Theorem (cf. [15]). The only known uniquely 3-edge-colorable non-planar graph is the generalized Petersen graph $G(9, 2)$, also called Tutte graph [17]. It has been conjectured that it is the only such graph (see Bollobás [2] and Schwenk [16]). A stronger version of Conjecture 1.5 for Petersen-minor free graphs was proposed in [19,20] as follows.

Conjecture 1.6 (*[19]*). *Let G be a Petersen-minor free cubic graph. If G is uniquely* 3*-edge-colorable, then G contains a triangle.*

Edmonds [4] proved that every bridgeless cubic graph has a family of perfect matchings such that every edge is covered by precise p members in the family where p is an integer that depends on the graph G. It is evident that, for a bridgeless cubic graph G with a forcing edge e, the integer $p = 1$. Hence such graphs must be 3-edge-colorable. Note that a cubic graph generated from K_4 by a series of $Y \to \Delta$-operations is uniquely 3-edge-colorable. Therefore, Theorem 1.2 shows that a 3-connected cubic graph with uniquely forced perfect matchings is uniquely 3-edge-colorable. The 3-connectivity is a necessary condition for cubic graphs to be uniquely 3-edge-colorable. A non-3-edge-connected cubic graph illustrated in Fig. 2 is not uniquely 3-edge-colorable.

If G is a bipartite cubic graph, Theorem 1.3 implies that G has no forcing edge. Hence, every edge e of G is contained in at least two distinct perfect matchings. Note that every perfect matching of a bipartite cubic graph G is contained in a 1-factorization of G, therefore a 3-edge-coloring. It follows that a bipartite cubic graph G is not uniquely 3-edge-colorable.

Corollary 1.7. *Every uniquely* 3*-edge-colorable cubic graph contains an odd cycle.*

Let G be a uniquely 3-edge-colorable cubic graph. By the above corollary, let k be the smallest size of odd cycles in G. Conjecture 1.6 asserts that if G is Petersen-minor free, then $k = 3$. In general, k cannot be less than 5 because of the generalized Petersen graph $G(9, 2)$ which has girth 5 [17]. It is interesting to ask if k is bounded above by a constant or even stronger $k = 5$ for all uniquely 3-edge-colorable cubic graphs.

2. Proof of Theorem 1.2

Let G be a graph with a perfect matching M. A *bridge* of a graph G is an edge whose deletion increases the number of the components G. A graph is *bridgeless* if it does not contain a bridge. A circuit (or a path) is *M-alternating* if its edges alternate between M and $E(G) \setminus M$. Before processing to prove our main result, we need some technical lemmas. The following lemma was first obtained by Kotzig [12] in matching version and later by Grossman and Häggkvist [8] in edge-coloring version, also Yeo [18]. This result is also proved by Lovász and Plummer by using the Cathedral Theorem (Theorem 5.3.8 in [13]). Here, we include a short proof for self-completeness.

Lemma 2.1. *Let G be a bridgeless graph with a perfect matching M. Then G has an M-alternating circuit.*

Proof. Let G be a minimum counterexample and P be a longest M-alternating path. Assume $P := x_0x_1 \cdots x_t$. It is obvious that both end edges $x_0x_1, x_{t-1}x_t \in M$. Since P is a longest M-alternating path, $N(x_0) \subseteq V(P)$. Let $x_i \in N(x_0) \setminus \{x_1\}$. The subscribe i must be even, for otherwise, $x_0 \cdots x_ix_0$ is an M-alternating circuit, which contradicts that G is a counterexample.

Let $X := x_0x_1 \cdots x_ix_0$. Then X is an odd circuit and $x_ix_{i+1} \in M$. Contract all edges of X. The new graph $G/E(X)$ is smaller and bridgeless. Note that $M \cap (E(G) \setminus E(X))$ is a perfect matching of $G/E(X)$. Since G is a minimum counterexample, $G/E(X)$ has an M-alternating circuit C. Then C contains the new vertex x, corresponding to the circuit X. Assume C passes through x by two edges e and e'. Since C is M alternating, one of e and e', say e', belongs to M, corresponding to x_ix_{i+1} in G. Assume that e is incident with x_j in X. Let $P' = x_jx_{j-1} \cdots x_0x_i$ if j is odd, and $P' = x_jx_{j+1} \cdots x_i$, otherwise. Replacing the vertex x of C by the path P' generates an M-alternating circuit of G, contradicting G is a counterexample. □

If a graph G with a perfect matching M has an M-alternating cycle C, then the symmetric difference $M \oplus E(C)$ is another perfect matching of G. So if M is uniquely forced by an edge e, then G has at most one M-alternating cycle which must contain the edge e. A graph G is *matching-covered* if every edge of G is contained by a perfect matching. The following is a well-known result (see 16.4.8 on Page 434 of [3]), a stronger version of Petersen Theorem [14].

Lemma 2.2. *Every bridgeless cubic multi-graph G is matching-covered.*

The following is a proposition for all graphs generated from K_4 by $Y \to \Delta$-operations.

Proposition 2.3. *Let G be a graph generated from K_4 via a series of $Y \to \Delta$-operations. If $G \neq K_4$, then G has at least two disjoint triangles.*

Proof. Assume that G is obtained from K_4 by k $Y \to \Delta$-operations and G_i is the graph obtained from K_4 by applying ith $Y \to \Delta$-operation in the process, for example $G_0 = K_4$ and $G_k = G$. Use induction on k. If $k = 1$, G is the 3-prism and has two disjoint triangles.

Now assume that the proposition holds for all G_i with $1 \leq i \leq k-1$. Without loss of generality, assume G_k is obtained from G_{k-1} by applying a $Y \to \Delta$-operation on a vertex x of G_{k-1}. Let $T_x = x_1x_2x_3x_1$ be the triangle obtained from the $Y \to \Delta$-operation on x.

By inductive hypothesis, G_{k-1} has two disjoint triangles T_1 and T_2. Since x belongs to at most one of these two triangles, say T_1, G_k has two disjoint triangles T_x and T_2. This completes the proof. □

Now, we are ready to prove our main result, Theorem 1.2.

Proof of Theorem 1.2. Let G be a minimum counterexample. Then G is a 3-connected cubic graph with a forcing edge $e = vv'$ which is not generated from K_4 by a series of $Y \to \Delta$-operations. Let M be the unique perfect matching of G containing e, and $G' = G \setminus \{v, v'\}$. Then G' is connected. Clearly, $M \setminus \{e\}$ is the unique perfect matching of G'. By Lemma 2.1, G' has a bridge. Let $e' = u_1u_2$ be a bridge of G'.

Deleting e' separates G' into two components G_1 and G_2. Assume that $u_i \in G_i$ for $i = 1, 2$. By parity, both G_1 and G_2 have odd number of vertices. It follows that $u_1u_2 \in M$.

Since G is 3-connected, both v and v' have neighbors in each G_i, say v_i and v_i' for $i = 1, 2$, respectively. Let $S_i = \{e', vv_i, v'v_i'\}$ for $i = 1, 2$. Then S_i is a 3-edge-cut of G. If both G_1 and G_2 have only one vertex, then G is K_4, contradicting that G is a counterexample. Without loss of generality, assume G_1 is not a singleton. Let

$$G_1' := G/E(G \setminus V(G_1)) \quad \text{and} \quad G_2' := G/E(G_1),$$

the graphs obtained from G by contracting all edges in $E(G - G_1)$ and $E(G_1)$ respectively. Then G_1' and G_2' are 3-connected cubic graphs. (See Fig. 3.) Let w_i be the new vertex in G_i' for $i = 1$ and 2. Since vv' is a forcing edge of G, it follows that G_1' has a unique perfect matching containing u_1w_1. Therefore, w_1u_1 is a forcing edge of G_1'.

Since G_1 is not a singleton, $4 \leq |V(G_1')| < |V(G)|$. Note that G_1' is a 3-connected cubic graph with a forcing edge u_1w_1, and G is a minimum counterexample. So G_1' is generated from K_4 by a series of $Y \to \Delta$-operations, and hence contains two disjoint triangles or $G_1' = K_4$. So G_1' contains a triangle $T = x_1x_2x_3x_1$ which does not contain w_1. Then T is also a triangle of G.

Claim. *The edge e is a forcing edge of $G/E(T)$.*

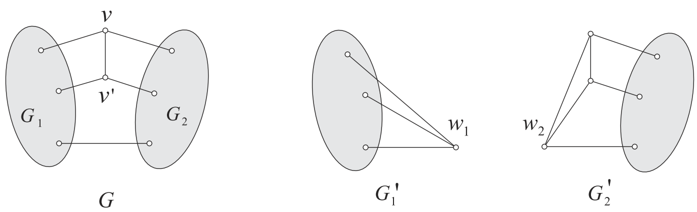

Fig. 3. 3-edge-cut reduction.

Proof of Claim. Since G is 3-connected, $G/E(T)$ is also 3-connected. By Lemma 2.2, $G/E(T)$ has a perfect matching N containing e. Suppose to the contrary that e is not a forcing edge of $G/E(T)$. Then $G/E(T)$ has an N-alternating cycle which does not contain e.

Let x be the new vertex obtained from contracting the edges of the triangle $T = x_1x_2x_3x_1$. Then N contains exactly one edge incident with x, say $e' = xy$ where $y \in G \setminus V(T)$. In the graph G, the edge e' joins y and a vertex of T, say x_1. Then $N' := N \cup \{x_2x_3\}$ is perfect matching of G containing e. If $x \notin V(C)$, then C is also an N'-alternating cycle of G, contradicting that e is a forcing edge of G. So suppose that $x \in V(C)$ and $yxz \subset C$. Let $C' := C \setminus \{x\} \cup \{yx_1x_2x_3z\}$ (relabel x_2 and x_3 if necessary). Then C' is an N'-alternating cycle of G which does not contain e neither, which contradicts that e is a forcing edge of G. This completes the proof of Claim.

By Claim, $G/E(T)$ is 3-connected cubic graph with a forcing edge e. Note that $|V(G/E(T))| < |V(G)|$ and G is a minimum counterexample. So $G/E(T)$ is generated from K_4 via a series of $Y \to \Delta$-operations. Since G can be obtained from $G/E(T)$ by applying an $Y \to \Delta$-operation on x, it follows that G can be generated from K_4 by a series of $Y \to \Delta$-operations. This completes the proof. □

A direct corollary of Theorem 1.2 and Proposition 2.3 is shown as follows.

Corollary 2.4. *Let G be a 3-connected cubic graph with a forcing edge. If $G \neq K_4$, then G contains two disjoint triangles.*

3. Proof of Theorem 1.3

The results in Section 2 demonstrate that a 3-connected cubic graphs with a forcing edge is either K_4 or contains two disjoint triangles. In the following, we consider cubic graphs with an edge-cut of size at most two.

Lemma 3.1. *Every connected cubic graph with a perfect matching has at least two distinct perfect matchings.*

Proof. Let G be a connected cubic graph with a perfect matching M. If G is bridgeless, Lemma 2.1 implies that G has an M-alternating circuit. Hence G has at least two distinct perfect matchings. So suppose that G has a bridge. Assume Q is an end-block and e is the bridge with one end-vertex v in Q. Then $M \cap E(Q)$ covers all vertices of Q, except v. Let $\overline{Q}$ be the cubic multi-graph obtained from Q by suppressing the vertex v: replacing two edges incident with v by one edge joining its two neighbors, denoted by e_v. Since G is simple, $\overline{Q}$ contains at least four vertices. Note that $\overline{Q}$ is 2-connected. By Lemma 2.2, let M' be a perfect matching of $\overline{Q}$ such that $e_v \notin M'$ and $M' \neq M \cap E(Q)$. Then $(M \setminus E(Q)) \cup M'$ is another perfect matching of G. □

The following lemma shows that a connected cubic graph with a forcing edge must be 2-connected.

Lemma 3.2. *If a cubic graph G is disconnected or has a bridge, then G has no forcing edges.*

Proof. If G has no perfect matchings, then the lemma holds trivially. So assume that G does have a perfect matching. If G is not connected, then G has at least two components. By Lemma 3.1, every component of G has two distinct perfect matchings. Hence G does not have a forcing edge. So, in the following, assume that G is connected and has a bridge.

Suppose to the contrary that G has a forcing edge e_0. Let M be the unique perfect matching of G containing e_0. Since G has a bridge, it follows that G has at least two end-blocks. Let $e = uv$ be a bridge separating G into two components Q_1 and Q_2 such that Q_1 is an end-block which does not contain e_0. Note that $e = e_0$ may hold. Without loss of generality, Assume that $u \in Q_1$ and $v \in Q_2$. Let $\overline{Q_1}$ be the cubic multi-graph obtained from Q_1 by suppressing the degree two vertex u and e_u be the new edge. Then $\overline{Q_1}$ is a bridgeless cubic graph. Let e_1 and e_2 be other two edges sharing a common end-vertex with e_u. By Lemma 2.2, $\overline{Q_1}$ has two distinct perfect matchings M_1 and M_2 such that $e_i \in M_i$ for $i = 1, 2$.

Note that $|V(Q_1)|$ is odd, and so is $|V(Q_2)|$. It follows that e belongs to all perfect matchings of G. Let $M' := M \cap E(Q_2)$. Then $M'_i := M' \cup M_i \cup \{uv\}$ is a perfect matching of G containing e_0 ($i = 1$ and 2). Note that M'_1 and M'_2 are distinct because M_1 and M_2 are distinct. Hence e_0 is contained by at least two distinct perfect matchings of G, contradicting that e_0 is a forcing edge. □

Proof of Theorem 1.3. Let G be a cubic graph with a forcing edge. By Lemma 3.2, G is 2-edge-connected. Use induction on the number of 2-edge-cut of G to show a slightly stronger result as follows:

($*$) A cubic graph G with a forcing edge has two disjoint triangles if $G \neq K_4$.

If G has no 2-edge-cut, then G is 3-connected and hence G has two disjoint triangles by Corollary 2.4. So in the following, assume that G has k 2-edge-cuts, and ($*$) holds for all cubic graphs with at most $k-1$ 2-edge-cuts.

Let $S = \{u_1v_1, u_2v_2\}$ be a 2-edge-cut of G. Deleting S from G separates G into two subgraphs G_1 and G_2. Without loss of generality, assume $u_1, u_2 \in V(G_1)$ and $v_1, v_2 \in V(G_2)$. Let $G_1' = G_1 \cup \{u_1u_2\}$ and $G_2' = G_2 \cup \{v_1v_2\}$. Then both G_1' and G_2' are cubic.

Claim. *Both G_1' and G_2' have a forcing edge.*

If not, assume that G_1' has no forcing edge; that is, for any edge of G_1', there are two distinct perfect matchings containing it.

Note that G has a forcing edge, say e. Let M be a unique perfect matching of G containing e.

First suppose that $M \cap S = \emptyset$, then $M_i = M \cap E(G_i)$ is a perfect matching of G_i' for $i = 1$ and 2. If $e \in M_1$, then G_1' has another perfect matching containing e, say M_1'. If M_1' does not contain u_1u_2, then $M_1' \cup M_2$ is another perfect matching of G containing of e, a contradiction. So M_1' contains u_1u_2. Now in G_2', let M_2' be the perfect matching of G_2' containing v_1v_2 by Lemma 2.2. Then $\big((M_1' \cup M_2') \setminus \{u_1u_2, v_1v_2\}\big) \cup \{u_1v_1, u_2v_2\}$ is another perfect matching of G containing e, a contradiction. So assume that $e \in M_2$. Let e' be the edge incident with u_1 contained in M_1. Since e' is not a forcing edge of G_1', it follows that G_1' has another perfect matching M_1' containing e'. Note that M_1' does not contain u_1u_2 and hence it is a perfect matching of G_1. Hence $M_1' \cup M_2$ is a perfect matching containing e but distinct to M, a contradiction.

So, in the following, suppose $M \cap S \neq \emptyset$. By parity, we further have $S \subset M$. If $e \in S$, then u_1u_2 is a forcing edge of G_1', a contradiction. So $e \in M \cap E(G_1)$ or $M \cap E(G_2)$. Let $M_1 := \big(M \cap E(G_1)\big) \cup \{u_1u_2\}$ and $M_2 := \big(M \cap E(G_2)\big) \cup \{v_1v_2\}$. Then M_i is a perfect matching of G_i' for $i = 1$ and 2. If $e \in M_2 \cap E(G_2)$, then let M_1' be another perfect matching containing u_1u_2 since u_1u_2 is not a forcing edge of G_1'. Then $(M_1' \cup M_2 \cup S) \setminus \{u_1u_2, v_1v_2\}$ is another perfect matching of G containing e, a contradiction. So assume that $e \in M_1 \cap E(G_1)$. Again, G_1' has another perfect matching containing e, say M_1'. If M_1' does not contain u_1u_2, then let M_2' be a perfect matching containing an edge of G_2 incident with v_1 by Lemma 2.2. Then it follows that $M_1' \cup M_2'$ is a perfect matching of G containing e, a contradiction. So assume that M_1' contains u_1u_2. Then $(M_1' \cup M_2 \cup S) \setminus \{u_1u_2, v_1v_2\}$ is another perfect matching of G containing e, a contradiction again. The contradiction completes the proof of the claim.

By Claim, both G_1' and G_2' have a forcing edge and have at most $k-1$ 2-edge-cuts. Hence, by induction hypothesis, G_i' has two disjoint triangles or $G_i' = K_4$ for $i = 1$ and 2. So G_1' has a triangle T_1 such that $u_1u_2 \notin E(T_1)$ and G_2' has a triangle T_2 such that $v_1v_2 \notin E(T_2)$. Then T_1 and T_2 are disjoint triangles of G. This completes the proof. □

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